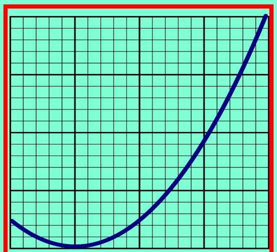
Math 125

Fall 2021

Lecture 43



$y = ax^2 + bx + c$

Class QZ 32

Solve:

$$3\left\{\;3x^2 + 4y^2 = 16\right.$$
$$4\left\{\;2x^2 - 3y^2 = 5\right.$$

$$\Rightarrow \begin{cases} 9x^2 + \cancel{12y^2} = 48 \\ 8x^2 - \cancel{12y^2} = 20 \end{cases}$$

$$17x^2 = 68$$

$$x^2 = 4 \qquad \boxed{x = \pm 2}\ \checkmark$$

$$3(4) + 4y^2 = 16$$

$$12 + 4y^2 = 16$$

$$4y^2 = 4 \qquad y^2 = 1 \qquad \boxed{y = \pm 1}\ \checkmark$$

$$\boxed{\{(2,1), (2,-1), (-2,1), (-2,-1)\}}$$

The difference of two numbers is 2.
$x \,\&\, y$
The sum of their squares is 10.
Find all such numbers.

$x - y = 2$

$x^2 + y^2 = 10$

$$\begin{cases} x - y = 2 \Rightarrow x = 2 + y \\ x^2 + y^2 = 10 \end{cases}$$

$$(2+y)^2 + y^2 = 10$$

$$(2+y)(2+y) + y^2 = 10$$

$$4 + 2y + 2y + y^2 + y^2 = 10$$

$$2y^2 + 4y + 4 - 10 = 0$$

$$2y^2 + 4y - 6 = 0$$

Divide by 2 to reduce

$$y^2 + 2y - 3 = 0$$

$$\to (y+3)(y-1) = 0$$

$y + 3 = 0$ $\quad$ $y - 1 = 0$

$y = -3$ $\quad$ $y = 1$

$x = 2 + (-3)$ $\quad$ $x = 2 + 1$

$x = -1$ $\quad$ $x = 3$

$$\boxed{\{(-1,-3), (3,1)\}}$$

The sum of two numbers is 5
$x \,\&\, y$
The difference of their squares is 5.
Find all such numbers.

$$\begin{cases} x + y = 5 \\ x^2 - y^2 = 5 \end{cases}$$

$x = 5 - y$

$$(5 - y)^2 - y^2 = 5$$

$$(5-y)(5-y) - y^2 = 5$$

$$25 - 5y - 5y + \cancel{y^2} - \cancel{y^2} = 5$$

$$25 - 10y = 5$$

$$\to -10y = 5 - 25$$

$$-10y = -20$$

$$\boxed{y = 2}$$

$$x = 5 - 2$$

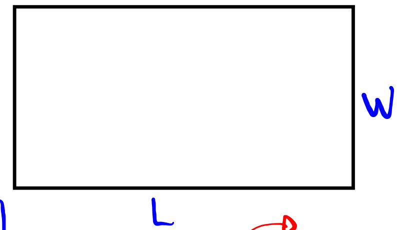
$$\boxed{x = 3}$$

$$\{(3,2)\}$$

Perimeter of a rectangle is 20 ft.

Area is 24 ft^2.

Find its dimensions.



$P = 20$

$2L + 2W = 20$

$\boxed{L + W = 10}$

$A = 24$

$\boxed{LW = 24}$

$$\begin{cases} L + W = 10 \rightarrow W = 10 - L \\ LW = 24 \rightarrow L(10 - L) = 24 \end{cases}$$

$10L - L^2 = 24$

$L^2 - 10L + 24 = 0$

$(L - 4)(L - 6) = 0$

$L = 4 \qquad L = 6$

$W = 6 \qquad W = 4$

Dimensions are 4 ft by 6 ft

Find dimensions of a rectangular garden with perimeter 40 m and area 96 m^2.

$$\div 2 \begin{cases} 2L + 2W = 40 \\ LW = 96 \end{cases} \qquad \begin{cases} L + W = 20 \\ LW = 96 \end{cases}$$

$W = 20 - L$

$L(20 - L) = 96$

$20L - L^2 = 96$

$L^2 - 20L + 96 = 0$

$(L - 12)(L - 8) = 0$

$L = 12 \qquad L = 8$

$W = 8 \qquad W = 12$

Dimensions are 8 m by 12 m.

Find two numbers $x \,\&\, y$ such that the sum of their squares is 25 and the difference of " " " 7.

$$\begin{cases} x^2 + y^2 = 25 \\ x^2 - y^2 = 7 \end{cases}$$

$2x^2 = 32$

$x^2 = 16$

$x = \pm 4$

$16 + y^2 = 25$

$y^2 = 9$

$\boxed{y = \pm 3}$

The numbers are

4, 3

4, −3

−4, 3

−4, −3

Solve

$$\begin{cases} -9x + y = 45 \\ y = x^3 + 5x^2 \end{cases}$$

$-9x + x^3 + 5x^2 = 45$

$x^3 + 5x^2 - 9x - 45 = 0$

$x^2(x+5) - 9(x+5) = 0$

$(x+5)(x^2-9) = 0$

$(x+5)(x+3)(x-3) = 0$

$\boxed{x=-5,\ y=0}$ $\boxed{x=-3,\ y=18}$ $\boxed{x=3,\ y=72}$

$y = 45 + 9x$

$y = 45 + 9(-5)$

$y = 0$

$y = 45 + 9(-3)$

$= 45 - 27$

$= 18$

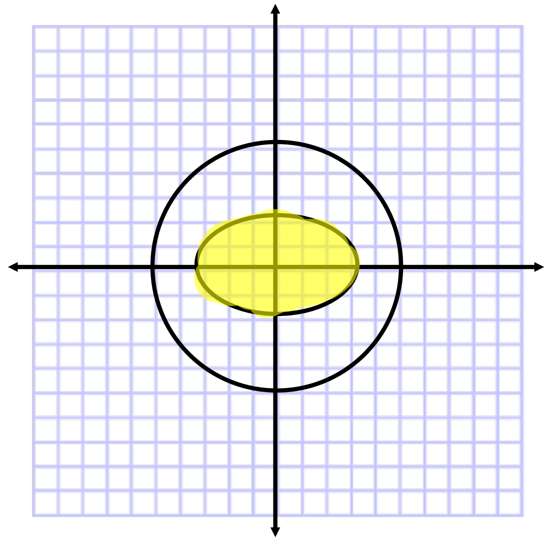
$y = 45 + 9(3)$

$= 45 + 27 = 72$

$\boxed{(-5,0),(-3,18),(3,72)}$

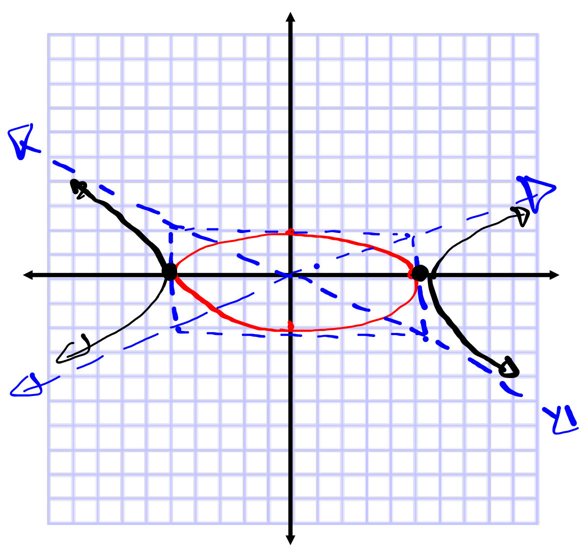
Graph & shade the overlap

$$\begin{cases} x^2 + y^2 = 25 \\ \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1 \end{cases}$$



Graph:

$$\begin{cases} \dfrac{x^2}{25} + \dfrac{y^2}{4} = 1 \\ \dfrac{x^2}{25} - \dfrac{y^2}{4} = 1 \end{cases}$$



Graph $(y-3)^2 - 4(x+4)^2 = 36$

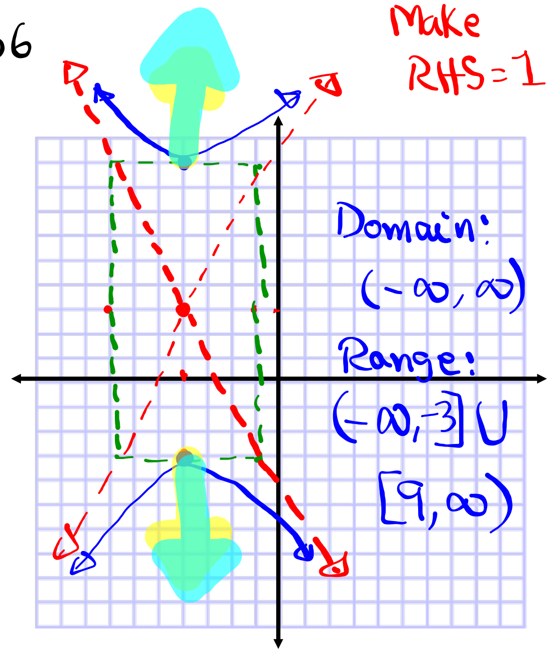
Divide by 36

$$\frac{(y-3)^2}{36} - \frac{(x+4)^2}{9} = 1$$

Center

$a^2 = 9 \qquad a = 3$

$b^2 = 36 \qquad b = 6$



Hint: Make RHS = 1

Domain: $(-\infty, \infty)$

Range: $(-\infty, -3] \cup [9, \infty)$

Rationalize the deno:

1) $$\frac{6}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{6\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\cancel{6}^{3}\sqrt[3]{4}}{\cancel{2}} = \boxed{3\sqrt[3]{4}}$$

Perfect cubes

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$\vdots$

2) $$\frac{1}{\sqrt[3]{25}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{\sqrt[3]{5}}{\sqrt[3]{125}} = \boxed{\frac{\sqrt[3]{5}}{5}}$$

Rationalize the deno

$$\frac{\sqrt{15}}{2\sqrt{5}+\sqrt{3}}\cdot\frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}-\sqrt{3}}$$

$$=\frac{2\sqrt{75}-\sqrt{45}}{4\sqrt{25}-\cancel{2\sqrt{15}}+\cancel{2\sqrt{15}}-\sqrt{9}}=\frac{2\sqrt{25}\sqrt{3}-\sqrt{9}\sqrt{5}}{4\cdot 5-3}=$$

$$=\frac{10\sqrt{3}-3\sqrt{5}}{17}$$

Class QZ 33

Solve

$$\begin{cases} x+y=1 & x=1-y \\ x^2+xy-y^2=-5 \end{cases}$$

Hint: Isolate one Variable, use Subs. Method. Final Ans in ordered-Pairs

$(1-y)^2+y(1-y)-y^2=-5$

$(1-y)(1-y)+y-y^2-y^2=-5$

$1-y-\cancel{y}+\cancel{y^2}+\cancel{y}-\cancel{y^2}-y^2=-5$

$1-y-y^2=-5$

$y^2+y-1-5=0$

$y^2+y-6=0$

$(y+3)(y-2)=0$

$y=-3 \quad y=2$

$x=1-y$

$x=1-(-3) \quad x=4$

$x=1-2 \quad x=-1$

$\boxed{\{(4,-3),(-1,2)\}}$